Επαναληπτικές 2019

Ενδεικτικές απαντήσεις

Θέμα Α

Α1 - β

Α2 - α

Α3 - δ

Α4 - β

Α5: $\Sigma - \Lambda - \Lambda - \Sigma - \Lambda$

Θέμα Β

Β1-(i)

$$x_1 = A \cdot \eta\mu(399\pi t) \quad (S.I.), \quad x_2 = A \cdot \eta\mu(401\pi t) \quad (S.I.)$$

$$\omega_1 = 399\pi \frac{rad}{s}$$

$$\omega_2 = 401\pi \frac{rad}{s}$$

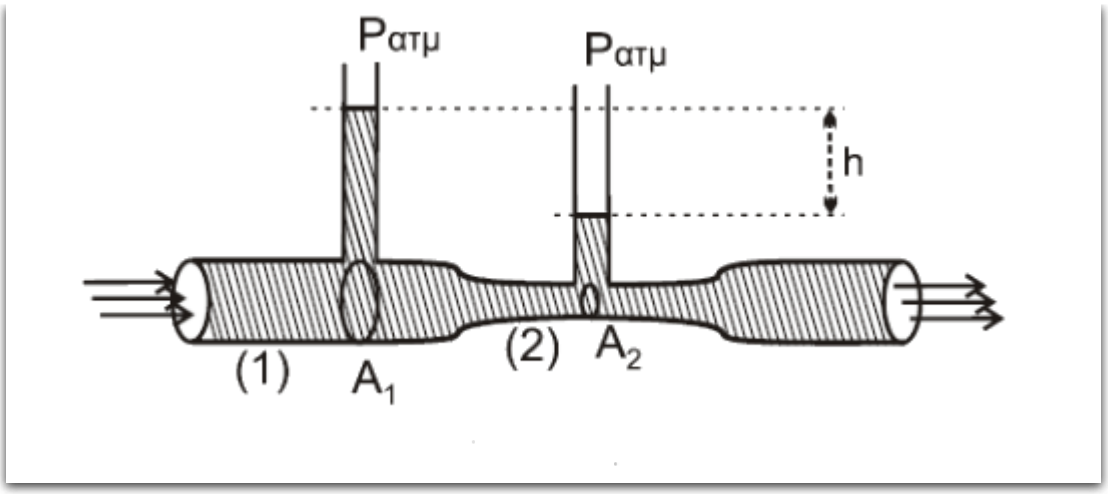
$$\omega_{\tau\epsilon\lambda} = \frac{\omega_1 + \omega_2}{2} = 400\pi \frac{rad}{s}, \quad T_{\tau\epsilon\lambda} = \frac{2\pi}{\omega_{\tau\epsilon\lambda}} = \frac{1}{200} sec$$

$$T_\delta = \frac{2\pi}{|\omega_1 - \omega_2|} = \frac{2\pi}{2\pi} = 1 sec$$

$$\sigma\epsilon \quad t = 2 \cdot T_\delta : \quad N_{\tau\alpha\lambda} = \frac{t}{T_{\tau\alpha\lambda}} = \frac{2 \cdot 1}{\frac{1}{200}} = 400 \tau\alpha\lambda\alpha\nu\tau\acute{\omega}\sigma\epsilon\iota\varsigma$$

άρα σωστό το i

Β2 - (iii)



πριν

$$\Pi_1 = \Pi_2 \Rightarrow A_1 \cdot \upsilon_1 = A_2 \cdot \upsilon_2 \overset{\frac{A_1}{A_2}=2}{\Longrightarrow} 2 \cdot \upsilon_1 = \upsilon_2$$

Εξίσωση Bernoulli για μια ρευματική γραμμή $(1 \rightarrow 2)$

$$P_1 + \frac{1}{2}\rho \cdot \upsilon_1^2 = P_2 + \frac{1}{2}\rho \cdot \upsilon_2^2 \Rightarrow P_{\alpha\tau\mu} + \rho \cdot g \cdot h_1 + \frac{1}{2}\rho \cdot \upsilon_1^2 = P_{\alpha\tau\mu} + \rho \cdot g \cdot h_2 + \frac{1}{2}\rho \cdot \upsilon_2^2$$

$$\rho \cdot g \cdot (h_1 - h_2) = \frac{3}{2} \cdot \rho \cdot \upsilon_1^2 \Rightarrow g \cdot h = \frac{3}{2} \cdot \upsilon_1^2$$

μετά

με ταχύτητα ροής $\upsilon_1' = 2\upsilon_1$ στην περιοχή (1) θα έχουμε

$$\Pi_1' = \Pi_2' \Rightarrow A_1 \cdot \upsilon_1' = A_2 \cdot \upsilon_2' \overset{\frac{A_1}{A_2}=2}{\Longrightarrow} 2 \cdot \upsilon_1' = \upsilon_2'$$

Εξίσωση Bernoulli για μια ρευματική γραμμή $(1' \rightarrow 2)$

$$P_1 + \frac{1}{2}\rho \cdot \upsilon_1'^2 = P_2 + \frac{1}{2}\rho \cdot \upsilon_2'^2 \Rightarrow P_{\alpha\tau\mu} + \rho \cdot g \cdot h_1' + \frac{1}{2}\rho \cdot \upsilon_1'^2 = P_{\alpha\tau\mu} + \rho \cdot g \cdot h_2' + \frac{1}{2}\rho \cdot \upsilon_2'^2$$

$$\rho \cdot g \cdot (h_1' - h_2') = \frac{3}{2} \cdot \rho \cdot \upsilon_1'^2 \Rightarrow g \cdot h' = \frac{3}{2} \cdot \upsilon_1'^2$$

$$\frac{gh}{gh'} = \frac{\frac{3}{2} \cdot \upsilon_1^2}{\frac{3}{2} \cdot \upsilon_1'^2} \Rightarrow \frac{h}{h'} = \frac{\upsilon_1^2}{\upsilon_1'^2} \overset{\upsilon_1'=2\upsilon_1}{\Longrightarrow} h' = 4h$$

άρα σωστό το *iii*

B3 - *(ii)*

πρίν την κρούση

$$p_1 = m \cdot \upsilon, \quad p_2 = 0$$

μετά την κρούση

$$p'_1 = m \cdot \upsilon_1 \quad p'_2 = m \cdot \upsilon_2$$

$$\Sigma \vec{F}_{\epsilon\xi} = 0 \iff \text{Α. Δ. Ο.} \quad \vec{p}_{\pi\rho\iota\nu} = \vec{p}_{\mu\epsilon\tau\alpha}$$

$$\vec{p_1} = \vec{p_1}' + \vec{p_2}'$$

$$p_1^2 = p_1'^2 + p_2'^2 + 2 \cdot p'_1 \cdot p'_2 \cdot \sigma\upsilon\nu\varphi \Rightarrow \upsilon^2 = \upsilon_1^2 + \upsilon_2^2 + 2 \cdot \upsilon_1 \cdot \upsilon_2 \cdot \sigma\upsilon\nu\varphi$$

Διατήρηση Κινητικής Ενέργειας

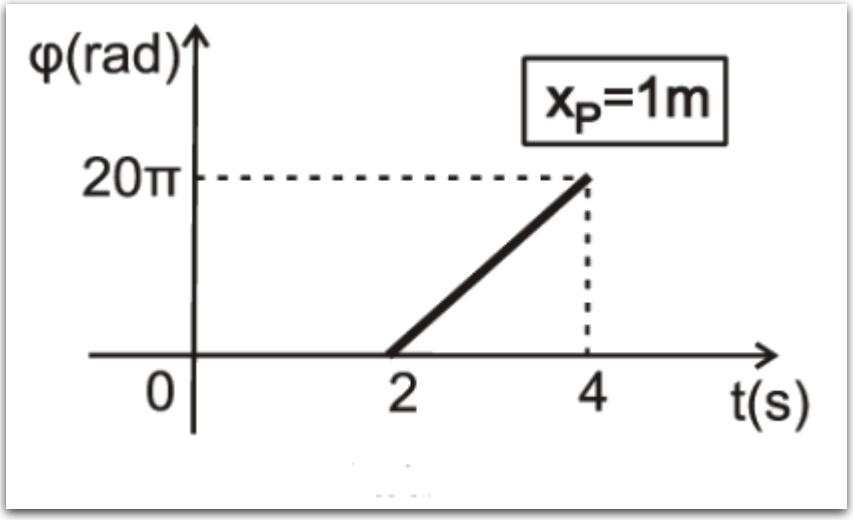
$$K_{\alpha\rho\chi} = K_{\tau\epsilon\lambda} \Rightarrow \frac{1}{2} \cdot m \cdot \upsilon^2 + 0 = \frac{1}{2} \cdot m \cdot \upsilon_1^2 + \frac{1}{2} \cdot m \cdot \upsilon_2^2 \Rightarrow \upsilon^2 = \upsilon_1^2 + \upsilon_2^2$$

οπότε από τις τελευταίες σχέσεις

$$2 \cdot \upsilon_1 \cdot \upsilon_2 \cdot \sigma\upsilon\nu\varphi = 0 \Rightarrow \sigma\upsilon\nu\varphi = 0 \Rightarrow \varphi = 90^o$$

άρα σωστό το *ii*

Θέμα Γ



$$y_0 = A \cdot \eta\mu\omega t$$

$$y = A \cdot \eta\mu(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda})$$

t=2sec

$$\varphi_\rho = 0 \Rightarrow \frac{2\pi \cdot 2}{T} - \frac{2\pi \cdot 1}{\lambda} = 0 \Rightarrow T = 2\lambda$$

t=4sec

$$\varphi_\rho = 20\pi \Rightarrow \frac{2\pi \cdot 4}{T} - \frac{2\pi \cdot 1}{\lambda} = 20\pi$$

$$\frac{4}{T}-\frac{1}{\lambda}=10\pi \Rightarrow \frac{4}{2\lambda}-\frac{1}{\lambda}=10\pi \Rightarrow \lambda=0,1m$$

$$T=0.2sec \quad \omega=\frac{2\pi}{0,2}=10\pi\frac{rad}{s}$$

Γ1

$$D=\Delta m\cdot\omega^2 \Rightarrow D=2\pi^2\cdot 10^{-4}\frac{N}{m}$$

$$E_T=\frac{1}{2}\cdot D\cdot A^2 \Rightarrow 16\pi^2\cdot 10^{-8}=\frac{1}{2}\cdot 2\pi^2\cdot 10^{-4}\cdot A^2 \Rightarrow A=0.04m$$

Γ2

$$y=0.04\cdot \eta\mu(\frac{2\pi t}{0.2}-\frac{2\pi x}{0.1}) \Rightarrow y=0.04\cdot \eta\mu(10\pi t-20\pi x) \quad (S.I.)$$

Γ3

$$\upsilon_\delta=\frac{\lambda}{T}=\frac{0.1}{0.2}=0.5\frac{m}{s}$$

$$t_\Sigma=\frac{x_\Sigma}{\upsilon_\delta}=\frac{1.15}{0.5}=2.3sec$$

$$y_P=0 \Rightarrow A\cdot \eta\mu\varphi_P \Rightarrow \varphi_P=2k\pi, \upsilon_P>0$$

$$\varphi_P=10\pi t-20\pi\cdot 1.15=10\pi t-23\pi \quad (S.I.)$$

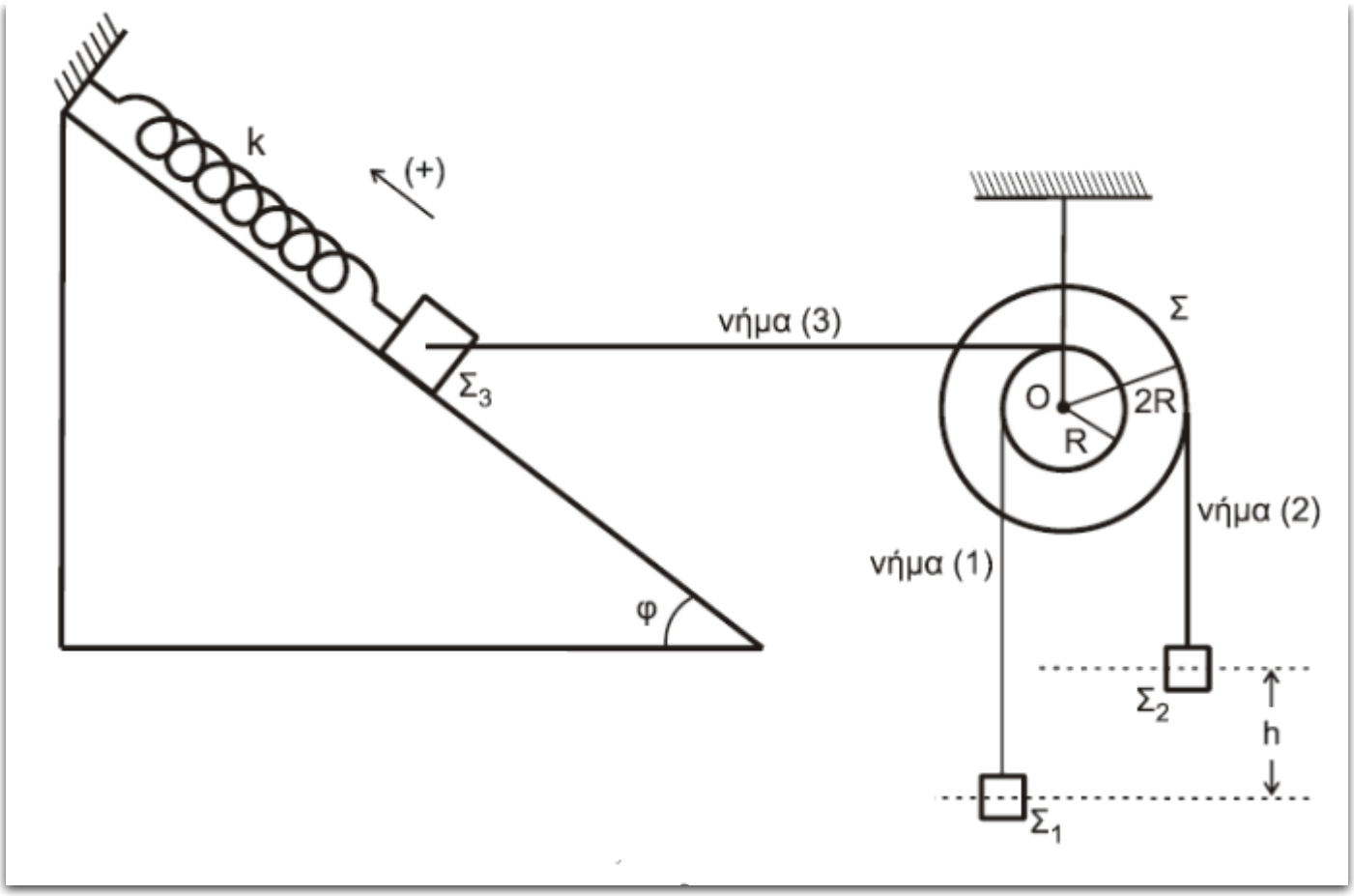
$$\varphi_P-\varphi_\Sigma=(10\pi t-20\pi)-(10\pi t-23\pi)=3\pi rad \overset{\varphi_P=2k\pi}{\Longrightarrow} \varphi_\Sigma=2k\pi-3\pi$$

$$\upsilon_\Sigma=\omega A\cdot \sigma\upsilon\nu\varphi_\Sigma=10\pi\cdot 0.04\cdot \sigma\upsilon\nu(2k\pi-3\pi)=0,4\pi\cdot(-1)-0.4\pi\frac{m}{s}$$

Γ4

$$y_\Sigma \begin{cases} 0 & 0\le t<2,3s \\ 0,04\cdot \eta\mu\,(10\pi t-23\pi), & t\ge 2.3sec \end{cases}$$

Θέμα Δ



Δ1

$$M_1, \quad \text{ισορροπία}, \quad \Rightarrow \Sigma\vec{F} = 0$$

$$T_1 = \mathrm{M}_1 \cdot g \Rightarrow T_1 = 20N$$

$$M_2, \quad \text{ισορροπία}, \quad \Rightarrow \Sigma\vec{F} = 0$$

$$T_2 = \mathrm{M}_2 \cdot g \Rightarrow T_2 = 15N$$

$$M_\Sigma, \quad \text{ισορροπία}, \quad \Rightarrow \Sigma\vec{\tau}_{(K)} = 0$$

$$T_2 \cdot 2R = T_1 \cdot R \cdot T_3 \cdot R \Rightarrow T_3 = 20N$$

$$M_3, \quad \text{ισορροπία}, \quad \Rightarrow \Sigma\vec{F} = 0$$

$$\Sigma F_x = 0 \Rightarrow F_{\varepsilon\lambda} - W_{3x} - T_{3x} = 0 \Rightarrow k \cdot \Delta l = \mathrm{M}_3 \cdot g \cdot \eta\mu\varphi + \mathrm{T}_3 \cdot \sigma\upsilon\nu\varphi \Rightarrow \Delta l = 0.12m$$

Δ2

$$D = k = M_3 \cdot \omega^2 \Rightarrow \omega = 10\frac{rad}{s}, \quad T = \frac{\pi}{5}sec$$

$$x = A \cdot \eta\mu(\omega t + \varphi_o)$$

$$t = 0, \quad \upsilon = 0 \quad x_\Gamma = -\mathrm{A}$$

$$\Theta.\,\mathrm{I}. \quad \Sigma F_x = 0 \Rightarrow F_{\varepsilon\lambda} - W_{3x} = 0 \Rightarrow k \cdot \Delta l_3 = M_3 \cdot g \cdot \eta\mu\varphi \Rightarrow \Delta l_3 = 0.08m$$

$$\mathrm{A} = \Delta l_1 - \Delta l_3 \Rightarrow A = 0.04m$$

$$t = 0 \Rightarrow -A = A \cdot \eta\mu\varphi_o \Rightarrow \eta\mu\varphi_o = \eta\mu\frac{3\pi}{2} \Rightarrow \varphi_o = 2k\pi + \frac{3\pi}{2} \Rightarrow \varphi_o = \frac{3\pi}{2}\frac{rad}{s}$$

$$t_1 = \frac{\pi}{15} sec x_1 = 0.04 \cdot \eta\mu(10 \cdot \frac{\pi}{15} + \frac{3\pi}{2}) \Rightarrow x_1 = 0.02m$$

$$\frac{dP}{dt} = \Sigma F_x = -D \cdot x \overset{t=t_1}{\Longrightarrow} \frac{dP}{dt} = -D \cdot x_1 \Rightarrow \frac{dP}{dt} = -6N$$

Δ3

$$\upsilon_1 = \omega \cdot R \Rightarrow a_1 = \alpha_{\gamma\omega\nu} \cdot R$$

$$\upsilon_2 = \omega \cdot 2R \Rightarrow a_2 = \alpha_{\gamma\omega\nu} \cdot 2R$$

$$\text{M}_2 \quad \Sigma F = m \cdot a \Rightarrow W_2 - T_2' = M_2 \cdot a_2 \Rightarrow 15 - T_2' = 1.5 \cdot \alpha_{\gamma\omega\nu} \cdot 0.2$$

$$\text{M} \quad \Sigma\tau = \text{I} \cdot \alpha_{\gamma\omega\nu} \Rightarrow T_2' \cdot 2R - T_1' \cdot R = 2M \cdot R^2 \cdot \alpha_{\gamma\omega\nu} \Rightarrow T_2' \cdot 2 - T_1' = 2 \cdot 1.5 \cdot 0.1 \cdot \alpha_{\gamma\omega\nu}$$

$$\text{M}_1 \quad \Sigma F = m \cdot a \Rightarrow \text{T}_1' - W_1 = M_1 \cdot a_1 \Rightarrow T_1' - 10 = 1 \cdot \alpha_{\gamma\omega\nu} \cdot 0.1$$

Λύση του συστήματος

$$\alpha_{\gamma\omega\nu} = 20 \frac{rad}{s^2}$$

Δ4

$$t_2: \quad h_1 = \theta \cdot R \quad h_2 = \theta \cdot 2R \overset{h_1+h_2=h}{\Longrightarrow} 3\theta \cdot R = h \Rightarrow \theta = \frac{h}{3R} = \frac{0.48}{0.3} \Rightarrow \theta = 1,6rad$$

$$\theta = \frac{1}{2} \cdot \alpha_{\gamma\omega\nu} t^2 \Rightarrow 1.6 = \frac{1}{2} \cdot 20 \cdot t_2^2 \Rightarrow t_2 = 0.4sec$$

$$\omega = \alpha_{\gamma\omega\nu} \cdot t_2 \Rightarrow \omega = 8 \frac{rad}{s}$$

$$L = I \cdot \alpha_{\gamma\omega\nu} = 2\text{M} \cdot R^2 \cdot \alpha_{\gamma\omega\nu} \Rightarrow L = 0.24kg \cdot \frac{m}{s^2}$$

Δ5

$$\text{N} = \frac{\theta_3}{2\pi} \Rightarrow \theta_3 = \frac{20}{\pi} \cdot 2\pi \Rightarrow \theta_3 = 40rad$$

$$\theta_3 = \frac{1}{2} \alpha_{\gamma\omega\nu} t_3^2 \Rightarrow t_3 = 2sec$$

$$\omega_3 = \alpha_{\gamma\omega\nu} t_3 \Rightarrow \omega_3 = 40 \frac{rad}{s}$$

$$\Sigma\tau = \text{I} \cdot \alpha_{\gamma\omega\nu} = 2M \cdot R^2 \cdot \alpha_{\gamma\omega\nu} \Rightarrow \Sigma\tau = 0,6N \cdot m$$

$$\Theta.\text{M.K.E.} \quad dK = dW \Rightarrow dK = \Sigma\tau \cdot d\theta \Rightarrow \frac{dK}{dt} = \Sigma\tau \cdot \frac{d\theta}{dt} \overset{t=t_3}{\Longrightarrow} \frac{dK}{dt} = 24 \frac{J}{s}$$

Μπορείτε να εκτυπώσετε τις λύσεις σε μορφή pdf από εδώ και τα θέματα από εδώ